



Dynamic event-triggered mechanism for H_∞ non-fragile state estimation of complex networks under randomly occurring sensor saturations[☆]

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ABSTRACT

In this paper, the problem of non-fragile H_∞ state estimation is investigated for a class of discrete-time complex networks subject to randomly occurring sensor saturations (ROSSs) under a dynamic event-triggered mechanism (DETM). The ROSS phenomenon is taken into account in the network measurements as a reflection of the probabilistic limitation of the physical sensors, and the DETM is implemented to govern the signal transmission from the sensor to its corresponding state estimator. The objective of the problem addressed is to design an H_∞ non-fragile state estimator under the DETM that can tolerate the possible gain perturbations, thereby possessing the desired non-fragility. By constructing a novel Lyapunov function, a sufficient condition is established such that the estimation error dynamics is exponentially mean-square stable with a prescribed H_∞ performance level, and then the estimator gains are parameterized according to certain matrix inequalities. A simulation example is provided to demonstrate the effectiveness of the proposed state estimation scheme.

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1. Introduction

As one of the fundamental issues in signal processing and network control areas, the state estimation (or filtering) problems have been attracting persistent research interests in the last few decades. A number of effective estimation schemes have been developed with a rich body of results available in the literature, see e.g. [5,15,20,22,24,31,47]. Roughly speaking, existing state estimation techniques can be divided into two categories: one is the popular Kalman estimation method whose main idea is to design an optimal state estimator to guarantee the minimum estimation error variance, and the other is the so-called H_∞ estimation scheme that aims to ensure the H_∞ performance requirement of the estimation error dynamics. It is worth mentioning that, compared to the Kalman estimation algorithm that relies on the assumptions

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of Gaussian-type noises with exactly known statistics, the H_∞ estimation strategy only requires the external disturbance to be energy-bounded. Accordingly, the H_∞ state estimation problem has become an attractive focus of research with wide application potentials, see e.g. [7,14,33,36,40,41] and the reference therein.

Complex networks (CNs) have proven to be an effective model for many real-world systems (e.g. Internet, scientific citation web, neural networks and cyber physical systems) that exhibit time-spatial characteristics. For decades, the dynamics analysis issues of CNs have been gaining an ever-increasing momentum from many research communities [1–3,8,9,28]. Among others, the state estimation problem for CNs has recently received particular research attention since it is often necessary to acquire accurate states of the underlying CNs so as to fulfill certain tasks such as scheduling, optimization and pinning control. To date, many excellent results have been published on the state estimation problem for various CNs, see e.g. [14,18,21,31]. On the other hand, sensor saturation is a frequently encountered phenomenon in reality that results mainly from the physical limitations of components, and such a phenomenon may occur in a random way because of the unsteady environment changes. In this context, the so-called randomly occurring sensor saturations (ROSSs) have aroused an increasing interest with respect to networked systems, see e.g. [19,36,40]. Nevertheless, when it comes to CNs with ROSSs, the corresponding state estimation problem has not been properly examined yet and deserves further investigation.

So far, for the state estimation issues of CNs, most relevant results have been obtained based on the implicit assumption that the designed estimator can be accurately implemented. Such an assumption is, unfortunately, a bit unrealistic since the estimator gain might be subjected to certain degree of perturbations during the implementation due to a variety of factors including the aging of the components, the finite word length in digital systems and the analogue-digital conversion. As such, it is naturally desired that the designed estimator is equipped with the non-fragility, that is, the estimation performance is insensitive to the implementation error with respect to the estimation gains. Therefore, in the last few years, much research attention has been devoted to the development of non-fragile techniques capable of tolerating the effects from estimator gain perturbations, and a large number of results have been reported in the literature, see e.g. [25,27,29,34,42,44,45]. For example, in [25,42,45], the additive gain perturbations have been tackled and the corresponding non-fragile state estimators have been designed. In [44], the multiplicative gain perturbations have been considered and the non-fragile state estimator design algorithm has been proposed.

On another research front, with the fast development of network communication technologies, many transmission strategies have been proposed for the purpose of avoiding unnecessary waste of limited network resources. Some widely used strategies include, but are not limited to, network scheduling protocols, static event-triggered mechanisms (SETMs) and dynamic event-triggered mechanisms (DETM), see e.g. [10,11,13,23,26,30,32,35,43,46,48]. Among these transmission methods, the DETM has been preferred in practice because of its capability of playing adequate tradeoff between the reduction of the communication burden and the preservation of the closed-loop stability [13]. As such, in recent years, many attempts have been made on the control/estimation problems under DETMs, see e.g. [12,13,16,17,37]. Nevertheless, as for CNs, the state estimation problem under DETMs has not been fully investigated yet, and this constitutes the main motivation of our current study.

Motivated by the above discussions, in this paper, our aim is to look into the H_∞ non-fragile state estimation issue for a kind of CNs subject to ROSSs under DETMs. The main contributions of this paper are listed as follows: 1) the CNs under consideration are quite comprehensive that takes DETMs, ROSSs and gain perturbations into simultaneous consideration; 2) the H_∞ non-fragile state estimator design issue is investigated, for the first time, for CNs under DETMs; and 3) state estimators are purposely designed with guaranteed stability, H_∞ performance and non-fragility concerning the estimation error dynamics. Finally, a simulation example is provided to illustrate the usefulness of the proposed state estimation algorithm.

Notation In this paper, \mathbb{R}^n and $\mathbb{R}^{n \times m}$ are used to denote, respectively, the n -dimensional Euclidean space and the set of all $n \times m$ real matrices. The symbol “ \otimes ” is the Kronecker product and $\|\cdot\|$ means the Euclidean norm. The superscript “ T ” represents matrix transposition and the asterisk “ $*$ ” in a matrix stands for the term induced by symmetry. The symbol $\text{diag}_n\{\dots\}$ describes a block-diagonal matrix $\text{diag}\{\underbrace{*, \dots, *}_n\}$. \mathbb{N} means the set of all nonnegative integers. 0 represents zero

matrix of compatible dimensions. I refers to the identity matrix and $\text{diag}\{\dots\}$ denotes a block-diagonal matrix. $l_2([0, \infty))$ is the space of square summable vector-valued functions. For real symmetric matrices X and Y , $X > Y$ ($X \geq Y$) means that $X - Y$ is positive definite (positive semi-definite). Matrices, if not specified explicitly, are assumed to have compatible dimensions.

2. Problem formulation

Consider a nonlinear CN with S coupled nodes described as follows:

$$\begin{cases} x_{i,k+1} = f(x_{i,k}) + \sum_{j=1}^S \omega_{ij} \Gamma x_{j,k} + B_i w_k, \\ z_{i,k} = E_i x_{i,k}, \quad i = 1, 2, \dots, S \end{cases} \quad (1)$$

where $x_{i,k} \in \mathbb{R}^{n_x}$ and $z_{i,k} \in \mathbb{R}^{n_z}$ are the state and the controlled output of the i th node, respectively. $W = (\omega_{ij})_{S \times S}$ is the coupled configuration matrix of the network whose elements satisfy $\omega_{ij} \geq 0$ ($i \neq j$) but not all zeros, and W is assumed to be symmetric with $\omega_{ii} = -\sum_{j=1, j \neq i}^S \omega_{ij}$. $\Gamma = \text{diag}\{l_1, l_2, \dots, l_{n_x}\}$ is an inner-coupling matrix. B_i and E_i are known constant matrices. $w_k \in \mathbb{R}^{n_w}$ denotes the exogenous disturbance belonging to $l_2[0, +\infty)$.

Assumption 1. The nonlinear function $f : \mathbb{R}^{n_x} \rightarrow \mathbb{R}^{n_x}$ in (1) is continuous, and satisfies $f(0) = 0$ and the following sector-bounded condition:

$$[f(x) - f(y) - U_1(x - y)]^T [f(x) - f(y) - U_2(x - y)] \leq 0, \tag{2}$$

for all $x, y \in \mathbb{R}^{n_x}$, where U_1 and U_2 are known matrices of appropriate dimensions.

The measurement output for the i th ($1 \leq i \leq S$) node is described as follows:

$$y_{i,k} = \pi_{i,k} \varrho(C_i x_{i,k}) + (1 - \pi_{i,k}) C_i x_{i,k} + D_i v_k \tag{3}$$

where $y_{i,k} \in \mathbb{R}^{n_y}$ is the measurement output from the i th node, C_i and D_i are known matrices, $v_k \in \mathbb{R}^{n_v}$ is the measurement noise belonging to $l_2[0, +\infty)$, and $\pi_{i,k}$ is a Bernoulli distributed variable taking values on 0 or 1 with the following probability distribution:

$$\text{Prob}\{\pi_{i,k} = 1\} = \alpha_i, \quad \text{Prob}\{\pi_{i,k} = 0\} = 1 - \alpha_i, \tag{4}$$

with $\alpha_i \in [0, 1]$. Furthermore, we assume that the random variables $\pi_{1,k}, \dots, \pi_{S,k}$ are mutually independent. The saturation function $\varrho(\cdot)$ is defined as

$$\varrho(r) \triangleq [\varrho(r_1) \quad \dots \quad \varrho(r_{n_y})]^T, \quad \forall r \in \mathbb{R}^{n_y} \tag{5}$$

where $\varrho(r_j) = \text{sign}(r_j) \min\{\kappa_j, |r_j|\}$. Here, ‘sign’ is the signum function and κ_j stands for the saturation level. From the above definition of saturation function, it can be verified that $\varrho(\cdot)$ satisfies

$$[\varrho(r_i) - \beta_i r_i]^T [\varrho(r_i) - r_i] \leq 0 \tag{6}$$

where β_i is a known constant satisfying $0 < \beta_i < 1$.

Remark 1. By introducing a set of Bernoulli distributed variables $\pi_{i,k}$ ($1 \leq i \leq S$), the actual measurement output modeled by (3) is capable of characterizing the phenomenon of ROSSs, where $\pi_{i,k} = 1$ means that the sensor i suffers from saturation, while $\pi_{i,k} = 0$ implies that the sensor i works normally.

For the sake of saving energy, a DETM is used on each node to determine if the measurement should be transmitted to the state estimator. For node i ($1 \leq i \leq S$), denote by $0 \leq t_0^i < t_1^i < \dots < t_l^i < \dots$ the transmission instants determined by the following condition:

$$t_{l+1}^i = \min \left\{ k \in \mathbb{N} \mid k > t_l^i, \frac{1}{\theta_i} \varsigma_{i,k} + \sigma_i y_{i,k}^T y_{i,k} - \phi_{i,k}^T \phi_{i,k} \leq 0 \right\} \tag{7}$$

where σ_i and θ_i are given positive scalars, $\phi_{i,k}$ is defined by $\phi_{i,k} \triangleq y_{i,k} - y_{i,t_l^i}$ with y_{i,t_l^i} being the transmitted measurement at latest event time, and $\varsigma_{i,k}$ is an internal dynamic variable that satisfies

$$\varsigma_{i,k+1} = \lambda_i \varsigma_{i,k} + \sigma_i y_{i,k}^T y_{i,k} - \phi_{i,k}^T \phi_{i,k}, \quad \varsigma_{i,0} = \varsigma_0^i \tag{8}$$

with $\lambda_i \in (0, 1)$ being a given constant and $\varsigma_0^i \geq 0$ being the given initial condition.

Remark 2. In the dynamic event-triggering condition (7), the internal dynamic variable $\varsigma_{i,k}$ is a key ingredient of the DETM, which can be deemed as a filtered version of the signal used to trigger events [13]. In practice, if the condition (7) is satisfied, then the sensor node would transmit its current measurement, namely, $y_{i,k}$, to its corresponding estimator. Furthermore, note that, letting $\theta_i \rightarrow +\infty$, the proposed DETM (7) reduces to the SETM $t_{l+1}^i = \min\{k \in \mathbb{N} \mid k > t_l^i, \sigma_i y_{i,k}^T y_{i,k} \leq \phi_{i,k}^T \phi_{i,k}\}$. Therefore, the DETM includes the SETM as a special case.

In terms of the dynamic event-triggered measurement, the i th ($1 \leq i \leq S$) state estimator is of the following form:

$$\begin{cases} \hat{x}_{i,k+1} = f(\hat{x}_{i,k}) + \sum_{j=1}^S \omega_{ij} \Gamma \hat{x}_{j,k} + (K_i + \Delta K_i) (y_{i,t_l^i} - C_i \hat{x}_{i,k}), \\ \hat{z}_{i,k} = E_i \hat{x}_{i,k} \end{cases} \tag{9}$$

for $k \in [t_l^i, t_{l+1}^i)$ ($l \geq 0$), where $\hat{x}_{i,k}$ is the estimate of state $x_{i,k}$, $\hat{z}_{i,k}$ is the estimate of controlled output $z_{i,k}$ and K_i is the estimator gain to be designed. ΔK_i is the gain perturbation satisfying

$$\Delta K_i \triangleq M_i F_{i,k} N_i \tag{10}$$

where M_i and N_i are known matrices, and $F_{i,k}$ is an unknown matrix satisfying $F_{i,k}^T F_{i,k} \leq I$.

Setting $e_{i,k} \triangleq x_{i,k} - \hat{x}_{i,k}$ as the state estimation error and $\tilde{z}_{i,k} \triangleq z_{i,k} - \hat{z}_{i,k}$ as the output estimation error, we obtain from (1) and (9) that

$$\begin{cases} e_{i,k+1} = \tilde{f}(e_{i,k}) + \sum_{j=1}^S \omega_{ij} \Gamma e_{j,k} + B_i w_k - (K_i + \Delta K_i) (y_{i,t_l^i} - C_i \hat{x}_{i,k}), \\ \tilde{z}_{i,k} = E_i e_{i,k} \end{cases} \tag{11}$$

for $k \in [t_l^i, t_{l+1}^i)$ ($l \geq 0$), where $\tilde{f}(e_{i,k}) \triangleq f(x_{i,k}) - f(\hat{x}_{i,k})$.

Noting the definition of $\phi_{i,k}$, (11) can be rewritten as follows:

$$\begin{cases} e_{i,k+1} = \tilde{f}(e_{i,k}) + \sum_{j=1}^S \omega_{ij} \Gamma e_{j,k} + B_i w_k - (K_i + \Delta K_i) (\pi_{i,k} \mathcal{Q}(C_i x_{i,k}) + (1 - \pi_{i,k}) \\ \quad \times C_i x_{i,k} + D_i v_k - \phi_{i,k} - C_i \hat{x}_{i,k}), \\ \tilde{z}_{i,k} = E_i e_{i,k} \end{cases} \tag{12}$$

For simplicity, we denote

$$\begin{aligned} \Delta K &\triangleq \text{diag}\{\Delta K_1, \dots, \Delta K_S\}, f(x_k) \triangleq [f^T(x_{1,k}) \quad \dots \quad f^T(x_{S,k})]^T, \tilde{f}(e_k) \triangleq [\tilde{f}^T(e_{1,k}) \quad \dots \quad \tilde{f}^T(e_{S,k})]^T, \\ x_k &\triangleq [x_{1,k}^T \quad \dots \quad x_{S,k}^T]^T, e_k \triangleq [e_{1,k}^T \quad \dots \quad e_{S,k}^T]^T, \phi_k \triangleq [\phi_{1,k}^T \quad \dots \quad \phi_{S,k}^T]^T, B \triangleq [B_1^T \quad \dots \quad B_S^T]^T, \\ \tilde{z}_k &\triangleq [\tilde{z}_{1,k}^T \quad \dots \quad \tilde{z}_{S,k}^T]^T, E \triangleq \text{diag}\{E_1, \dots, E_S\}, D \triangleq [D_1^T \quad \dots \quad D_S^T]^T, C \triangleq \text{diag}\{C_1, \dots, C_S\}, \\ \bar{\mathcal{Q}}(Cx_k) &\triangleq [\varrho^T(C_1 x_{1,k}) \quad \dots \quad \varrho^T(C_S x_{S,k})]^T, K \triangleq \text{diag}\{K_1, \dots, K_S\}, M \triangleq \text{diag}\{M_1, \dots, M_S\}, \\ F_k &\triangleq \text{diag}\{F_{1,k}, \dots, F_{S,k}\}, N \triangleq \text{diag}\{N_1, \dots, N_S\}. \end{aligned}$$

Letting $\bar{e}_k \triangleq [x_k^T \quad e_k^T]^T$ and $\vartheta_k \triangleq [w_k^T \quad v_k^T]^T$, we derive the following augmented system:

$$\begin{cases} \bar{e}_{k+1} = F(\bar{e}_k) + \bar{K}_2 \phi_k - \bar{K}_2 \Theta \bar{\mathcal{Q}}(\bar{C} \bar{e}_k) - \sum_{i=1}^S \tilde{\pi}_{i,k} \bar{K}_2 \Pi_i \bar{\mathcal{Q}}(\bar{C} \bar{e}_k) \\ \quad + \sum_{i=1}^S \tilde{\pi}_{i,k} \bar{K}_2 \Pi_i \bar{C} \bar{e}_k + \bar{K}_1 \bar{e}_k + \bar{B} \vartheta_k, \\ \tilde{z}_k = \bar{E} \bar{e}_k \end{cases} \tag{13}$$

where

$$\begin{aligned} \Pi_i &\triangleq \text{diag}\{\underbrace{0 \dots 0}_{i-1}, I, \underbrace{0 \dots 0}_{S-i}\}, \bar{C} \triangleq [C \quad 0], \bar{E} \triangleq [0 \quad E], \tilde{\pi}_{i,k} \triangleq \pi_{i,k} - \alpha_i, \\ \bar{K}_2 &\triangleq \begin{bmatrix} 0 \\ K + \Delta K \end{bmatrix}, \Theta \triangleq \text{diag}\{\alpha_1 I, \dots, \alpha_S I\}, \bar{K}_1 \triangleq \begin{bmatrix} W \otimes \Gamma & 0 \\ (K + \Delta K) \Theta C & W \otimes \Gamma - (K + \Delta K) C \end{bmatrix}, \\ F(\bar{e}_k) &\triangleq [f^T(x_k) \quad \tilde{f}^T(e_k)]^T, \bar{B} \triangleq \begin{bmatrix} B \\ B \\ - (K + \Delta K) D \end{bmatrix}. \end{aligned}$$

The objective of this paper is to design a set of non-fragile state estimators of the structure (9) for the CNs (1) such that the following requirements are satisfied.

- a) The augmented system (13) with $\vartheta_k = 0$ is exponentially stable in the mean square.
- b) Under the zero-initial condition, the output estimation error $\tilde{z}(k)$ satisfies

$$\mathbb{E} \left\{ \sum_{k=0}^{+\infty} \|\tilde{z}_k\|^2 \right\} < \gamma^2 \sum_{k=0}^{+\infty} \|\vartheta_k\|^2 \tag{14}$$

for all nonzero ϑ_k , where $\gamma > 0$ is the given attenuation level.

3. Main results

In this section, a sufficient condition is first derived under which both the exponential stability and the H_∞ performance constraint (14) are ensured for the estimation error dynamics with ROSSs and DETMs, and then the desired H_∞ non-fragile estimator is designed by solving a certain linear matrix inequality (LMI).

Before presenting our main results, the following lemmas are introduced.

Lemma 1. For the considered DETM (7)-(8) with $\varsigma_0^i \geq 0$ ($1 \leq i \leq S$), if the parameters λ_i and θ_i satisfy $\lambda_i \theta_i \geq 1$, then the internal dynamic variable $\varsigma_{i,k}$ satisfies $\varsigma_{i,k} \geq 0$ for all time.

Proof. For all $k \geq 0$, the dynamic event-triggering condition (7) ensures that

$$\sigma_i v_{i,k}^T y_{i,k} - \phi_{i,k}^T \phi_{i,k} \geq -\frac{1}{\theta_i} \varsigma_{i,k}.$$

In addition, it follows from (8) that

$$\varsigma_{i,k+1} \geq \left(\lambda_i - \frac{1}{\theta_i}\right) \varsigma_{i,k} \geq \dots \geq \left(\lambda_i - \frac{1}{\theta_i}\right)^{k+1} \varsigma_0^i.$$

Then, under the conditions $\lambda_i \theta_i \geq 1$ and $\zeta_0^i \geq 0$, it is easily seen that $\zeta_{i,k} \geq 0$ for all time. The proof is complete now. \square

Lemma 2 [38]. Let $Y = Y^T$, T , F and N be real matrices, where F satisfies $FF^T \leq I$. Then, $Y + TFN + (TFN)^T < 0$ holds if and only if there exists a positive scalar ϕ such that

$$Y + \phi^{-1}TT^T + \phi N^T N < 0.$$

Theorem 1. Let the disturbance level γ and the estimator gain K be given. The augmented system (13) (with $\vartheta_k = 0$) is exponentially stable in the mean square and the output estimation error \tilde{z}_k satisfies the H_∞ performance constraint (14) (for all nonzero ϑ_k) if there exist a positive definite matrix P , and three positive scalars μ_1, μ_2 and μ_3 satisfying

$$\tilde{\Omega} \triangleq \begin{bmatrix} \Omega_4 & \tilde{G}^T \tilde{P} & \tilde{A}^T P \\ * & -\tilde{P} & 0 \\ * & * & -P \end{bmatrix} < 0 \tag{15}$$

where

$$\begin{aligned} \tilde{U}_1 &\triangleq \text{diag}_2 \left\{ I_S \otimes \frac{U_1^T U_2 + U_2^T U_1}{2} \right\}, \tilde{U}_2 \triangleq -\text{diag}_2 \left\{ I_S \otimes \frac{U_1^T + U_2^T}{2} \right\}, \tilde{D} \triangleq [0 \quad D], \tilde{\alpha}_i \triangleq \alpha_i(1 - \alpha_i), \\ \tilde{\Lambda}_1 &\triangleq \text{diag} \left\{ \left(\frac{1}{\theta_1} + \mu_2 \right) \sigma_1 I, \dots, \left(\frac{1}{\theta_S} + \mu_2 \right) \sigma_S I \right\}, \tilde{\Lambda}_2 \triangleq \text{diag} \left\{ \left(\frac{1}{\theta_1} + \mu_2 \right) I, \dots, \left(\frac{1}{\theta_S} + \mu_2 \right) I \right\}, \\ \tilde{\Lambda}_3 &\triangleq \text{diag} \left\{ \frac{\lambda_1 - 1 + \mu_2}{\theta_1} I, \dots, \frac{\lambda_S - 1 + \mu_2}{\theta_S} I \right\}, \tilde{\Psi} \triangleq \text{diag} \{ \beta_1, \dots, \beta_{n_y S} \}, \tilde{P} \triangleq \text{diag}_S \{ P \}, \\ \tilde{\Upsilon}_{11} &\triangleq -P + \sum_{i=1}^S \tilde{\alpha}_i \left(\Pi_i \tilde{C} \right)^T \tilde{\Lambda}_1 \left(\Pi_i \tilde{C}_k \right) - \mu_1 \tilde{U}_1 + \tilde{C}^T (I - \Theta) \tilde{\Lambda}_1 (I - \Theta) \tilde{C} + \tilde{E}^T \tilde{E} - \mu_3 \tilde{C}^T \tilde{\Psi}^T \tilde{C}, \\ \tilde{\Upsilon}_{13} &\triangleq -\sum_{i=1}^S \tilde{\alpha}_i \left(\Pi_i \tilde{C} \right)^T \tilde{\Lambda}_1 \Pi_i + \tilde{C}^T (I - \Theta) \tilde{\Lambda}_1 \Theta + \mu_3 \frac{\tilde{C}^T \tilde{\Psi}^T + \tilde{C}^T}{2}, \tilde{\Upsilon}_{36} \triangleq \Theta \tilde{\Lambda}_1 \tilde{D}, \\ \tilde{\Upsilon}_{33} &\triangleq -\mu_3 I + \sum_{i=1}^S \tilde{\alpha}_i \Pi_i \tilde{\Lambda}_1 \Pi_i + \Theta \tilde{\Lambda}_1 \Theta, \tilde{\Upsilon}_{66} \triangleq \tilde{D}^T \tilde{\Lambda}_1 \tilde{D} - \gamma^2 I, \tilde{\Upsilon}_{16} \triangleq \tilde{C}^T (I - \Theta) \tilde{\Lambda}_1 \tilde{D}, \\ \tilde{A} &\triangleq [\tilde{K}_1 \quad I \quad -\tilde{K}_2 \Theta \quad \tilde{K}_2 \quad 0 \quad \tilde{B}], \tilde{G} \triangleq [\tilde{G}_1^T \tilde{C} \quad 0 \quad -\tilde{G}_1^T \quad 0 \quad 0 \quad 0], \\ \Omega_4 &\triangleq \begin{bmatrix} \tilde{\Upsilon}_{11} & -\mu_1 \tilde{U}_2 & \tilde{\Upsilon}_{13} & 0 & 0 & \tilde{\Upsilon}_{16} \\ * & -\mu_1 I & 0 & 0 & 0 & 0 \\ * & * & \tilde{\Upsilon}_{33} & 0 & 0 & \tilde{\Upsilon}_{36} \\ * & * & * & -\tilde{\Lambda}_2 & 0 & 0 \\ * & * & * & * & \tilde{\Lambda}_3 & 0 \\ * & * & * & * & * & \tilde{\Upsilon}_{66} \end{bmatrix}, G_1 \triangleq [\sqrt{\alpha_1} \Pi_1 \tilde{K}_2^T \quad \dots \quad \sqrt{\alpha_S} \Pi_S \tilde{K}_2^T]. \end{aligned}$$

Proof. Define the following Lyapunov function:

$$J_k \triangleq \tilde{e}_k^T P \tilde{e}_k + \sum_{i=1}^S \frac{1}{\theta_i} \zeta_{i,k}. \tag{16}$$

Then, it is calculated that

$$\begin{aligned} &\mathbb{E}\{\Delta J_k\} \\ &= \mathbb{E}\{J_{k+1} - J_k\} \\ &= \mathbb{E}\left\{ \tilde{e}_{k+1}^T P \tilde{e}_{k+1} - \tilde{e}_k^T P \tilde{e}_k + \sum_{i=1}^S \frac{1}{\theta_i} (\zeta_{i,k+1} - \zeta_{i,k}) \right\}. \end{aligned} \tag{17}$$

It follows from (7) and (13) with $\vartheta_k = 0$ that

$$\begin{aligned} &\mathbb{E}\{\Delta J_k\} \\ &= \mathbb{E}\left\{ \left(F(\tilde{e}_k) + \tilde{K}_1 \tilde{e}_k + \tilde{K}_2 \phi_k - \tilde{K}_2 \Theta \tilde{Q}(\tilde{C} \tilde{e}_k) - \sum_{i=1}^S \tilde{\pi}_{i,k} \tilde{K}_2 \Pi_i \tilde{Q}(\tilde{C} \tilde{e}_k) + \sum_{i=1}^S \tilde{\pi}_{i,k} \tilde{K}_2 \Pi_i \tilde{C} \tilde{e}_k \right)^T \right. \\ &\quad \times P \left(F(\tilde{e}_k) + \tilde{K}_1 \tilde{e}_k + \tilde{K}_2 \phi_k - \tilde{K}_2 \Theta \tilde{Q}(\tilde{C} \tilde{e}_k) - \sum_{i=1}^S \tilde{\pi}_{i,k} \tilde{K}_2 \Pi_i \tilde{Q}(\tilde{C} \tilde{e}_k) + \sum_{i=1}^S \tilde{\pi}_{i,k} \tilde{K}_2 \Pi_i \tilde{C} \tilde{e}_k \right) \\ &\quad \left. + \sum_{i=1}^S \frac{1}{\theta_i} \left((\lambda_i - 1) \zeta_{i,k} + \sigma_i y_{i,k}^T y_{i,k} - \phi_{i,k}^T \phi_{i,k} \right) - \tilde{e}_k^T P \tilde{e}_k \right\} \end{aligned}$$

$$\begin{aligned}
 &= \mathbb{E} \left\{ \left(F(\bar{e}_k) + \bar{K}_1 \bar{e}_k + \bar{K}_2 \phi_k - \bar{K}_2 \Theta \bar{\varrho}(\bar{C}\bar{e}_k) \right)^T P \left(F(\bar{e}_k) + \bar{K}_1 \bar{e}_k + \bar{K}_2 \phi_k - \bar{K}_2 \Theta \bar{\varrho}(\bar{C}\bar{e}_k) \right) \right. \\
 &\quad - 2 \sum_{i=1}^S \tilde{\pi}_{i,k} \left(\bar{K}_2 \Pi_i \bar{\varrho}(\bar{C}\bar{e}_k) \right)^T P \left(F(\bar{e}_k) + \bar{K}_1 \bar{e}_k + \bar{K}_2 \phi_k - \bar{K}_2 \Theta \bar{\varrho}(\bar{C}\bar{e}_k) \right) + 2 \sum_{i=1}^S \tilde{\pi}_{i,k} \left(\bar{K}_2 \Pi_i \bar{C}\bar{e}_k \right)^T \\
 &\quad \times P \left(F(\bar{e}_k) + \bar{K}_1 \bar{e}_k + \bar{K}_2 \phi_k - \bar{K}_2 \Theta \bar{\varrho}(\bar{C}\bar{e}_k) \right) - 2 \sum_{i=1}^S \sum_{j=1}^S \tilde{\pi}_{i,k} \tilde{\pi}_{j,k} \left(\bar{K}_2 \Pi_i \bar{\varrho}(\bar{C}\bar{e}_k) \right)^T P \left(\bar{K}_2 \Pi_j \bar{C}\bar{e}_k \right) \\
 &\quad + \sum_{i=1}^S \sum_{j=1}^S \tilde{\pi}_{i,k} \tilde{\pi}_{j,k} \left(\bar{K}_2 \Pi_i \bar{\varrho}(\bar{C}\bar{e}_k) \right)^T P \left(\bar{K}_2 \Pi_j \bar{\varrho}(\bar{C}\bar{e}_k) \right) + \sum_{i=1}^S \sum_{j=1}^S \tilde{\pi}_{i,k} \tilde{\pi}_{j,k} \left(\bar{K}_2 \Pi_i \bar{C}\bar{e}_k \right)^T P \left(\bar{K}_2 \Pi_j \bar{C}\bar{e}_k \right) \\
 &\quad + \sum_{i=1}^S \frac{1}{\theta_i} (\lambda_i - 1) \zeta_{i,k} + \left(\Theta \bar{\varrho}(\bar{C}\bar{e}_k) + (I - \Theta) \bar{C}\bar{e}_k \right)^T \Lambda_1 \left(\Theta \bar{\varrho}(\bar{C}\bar{e}_k) + (I - \Theta) \bar{C}\bar{e}_k \right) - \phi_k^T \Lambda_2 \phi_k \\
 &\quad + 2 \sum_{i=1}^S \tilde{\pi}_{i,k} \left(\Pi_i \bar{\varrho}(\bar{C}\bar{e}_k) \right)^T \Lambda_1 \left(\Theta \bar{\varrho}(\bar{C}\bar{e}_k) + (I - \Theta) \bar{C}\bar{e}_k \right) - 2 \sum_{i=1}^S \tilde{\pi}_{i,k} \left(\Pi_i \bar{C}\bar{e}_k \right)^T \Lambda_1 \left(\Theta \bar{\varrho}(\bar{C}\bar{e}_k) \right. \\
 &\quad \left. + (I - \Theta) \bar{C}\bar{e}_k \right) - 2 \sum_{i=1}^S \sum_{j=1}^S \tilde{\pi}_{i,k} \tilde{\pi}_{j,k} \left(\Pi_i \bar{\varrho}(\bar{C}\bar{e}_k) \right)^T \Lambda_1 \left(\Pi_j \bar{C}\bar{e}_k \right) + \sum_{i=1}^S \sum_{j=1}^S \tilde{\pi}_{i,k} \tilde{\pi}_{j,k} \left(\Pi_i \bar{\varrho}(\bar{C}\bar{e}_k) \right)^T \Lambda_1 \\
 &\quad \times \left. \left(\Pi_j \bar{\varrho}(\bar{C}\bar{e}_k) \right) + \sum_{i=1}^S \sum_{j=1}^S \tilde{\pi}_{i,k} \tilde{\pi}_{j,k} \left(\Pi_i \bar{C}\bar{e}_k \right)^T \Lambda_1 \left(\Pi_j \bar{C}\bar{e}_k \right) - \bar{e}_k^T P \bar{e}_k \right\} \tag{18}
 \end{aligned}$$

with

$$\Lambda_1 \triangleq \text{diag} \left\{ \frac{\sigma_1}{\theta_1} I, \dots, \frac{\sigma_S}{\theta_S} I \right\}, \quad \Lambda_2 \triangleq \text{diag} \left\{ \frac{1}{\theta_1} I, \dots, \frac{1}{\theta_S} I \right\}.$$

By noting the facts that $\mathbb{E}\{\tilde{\pi}_{i,k}\} = 0$, $\mathbb{E}\{\tilde{\pi}_{i,k}^2\} = \bar{\alpha}_i$ and $\mathbb{E}\{\tilde{\pi}_{i,k} \tilde{\pi}_{j,k}\} = 0$ (for $i \neq j$), it can be further obtained that

$$\begin{aligned}
 &\mathbb{E}\{\Delta J_k\} \\
 &= \mathbb{E} \left\{ \left(F(\bar{e}_k) + \bar{K}_1 \bar{e}_k + \bar{K}_2 \phi_k - \bar{K}_2 \Theta \bar{\varrho}(\bar{C}\bar{e}_k) \right)^T P \left(F(\bar{e}_k) + \bar{K}_1 \bar{e}_k - \bar{K}_2 \Theta \bar{\varrho}(\bar{C}\bar{e}_k) + \bar{K}_2 \phi_k \right) \right. \\
 &\quad + \sum_{i=1}^S \bar{\alpha}_i \left(\bar{K}_2 \Pi_i \bar{C}\bar{e}_k \right)^T P \left(\bar{K}_2 \Pi_i \bar{C}\bar{e}_k \right) + \sum_{i=1}^S \bar{\alpha}_i \left(\bar{K}_2 \Pi_i \bar{\varrho}(\bar{C}\bar{e}_k) \right)^T P \left(\bar{K}_2 \Pi_i \bar{\varrho}(\bar{C}\bar{e}_k) \right) \\
 &\quad - 2 \sum_{i=1}^S \bar{\alpha}_i \left(\bar{K}_2 \Pi_i \bar{\varrho}(\bar{C}\bar{e}_k) \right)^T P \left(\bar{K}_2 \Pi_i \bar{C}\bar{e}_k \right) - \bar{e}_k^T P \bar{e}_k - \phi_k^T \Lambda_2 \phi_k + \bar{\zeta}_k^T \Lambda_3 \bar{\zeta}_k \\
 &\quad + \left(\Theta \bar{\varrho}(\bar{C}\bar{e}_k) + (I - \Theta) \bar{C}\bar{e}_k \right)^T \Lambda_1 \left(\Theta \bar{\varrho}(\bar{C}\bar{e}_k) + (I - \Theta) \bar{C}\bar{e}_k \right) \\
 &\quad + \sum_{i=1}^S \bar{\alpha}_i \left(\Pi_i \bar{\varrho}(\bar{C}\bar{e}_k) \right)^T \Lambda_1 \left(\Pi_i \bar{\varrho}(\bar{C}\bar{e}_k) \right) + \sum_{i=1}^S \bar{\alpha}_i \left(\Pi_i \bar{C}\bar{e}_k \right)^T \Lambda_1 \left(\Pi_i \bar{C}\bar{e}_k \right) \\
 &\quad \left. - 2 \sum_{i=1}^S \bar{\alpha}_i \left(\Pi_i \bar{\varrho}(\bar{C}\bar{e}_k) \right)^T \Lambda_1 \left(\Pi_i \bar{C}\bar{e}_k \right) \right\} \\
 &= \mathbb{E} \left\{ \bar{\zeta}_k^T \left(\Omega_1 + \bar{A}^T P \bar{A} + \bar{G}^T \bar{P} \bar{G} \right) \bar{\zeta}_k \right\} \tag{19}
 \end{aligned}$$

where

$$\begin{aligned}
 \Lambda_3 &\triangleq \text{diag} \left\{ \frac{\lambda_1 - 1}{\theta_1} I, \dots, \frac{\lambda_S - 1}{\theta_S} I \right\}, \quad \zeta_k \triangleq \left[\bar{e}_k^T \quad F^T(\bar{e}_k) \quad \bar{\varrho}^T(\bar{C}\bar{e}_k) \quad \phi_k^T \quad \bar{\zeta}_k^T \right]^T, \quad \bar{\zeta}_k \triangleq \left[\zeta_{1,k}^{\frac{1}{2}} \quad \dots \quad \zeta_{S,k}^{\frac{1}{2}} \right]^T, \\
 \Upsilon_{11} &\triangleq -P + \sum_{i=1}^S \bar{\alpha}_i \left(\Pi_i \bar{C} \right)^T \Lambda_1 \left(\Pi_i \bar{C} \right) + \bar{C}^T (I - \Theta) \Lambda_1 (I - \Theta) \bar{C}, \quad \Upsilon_{33} \triangleq \Theta \Lambda_1 \Theta + \sum_{i=1}^S \bar{\alpha}_i \Pi_i \Lambda_1 \Pi_i, \\
 \Upsilon_{13} &\triangleq \bar{C}^T (I - \Theta) \Lambda_1 \Theta - \sum_{i=1}^S \bar{\alpha}_i \left(\Pi_i \bar{C} \right)^T \Lambda_1 \Pi_i, \quad \bar{A} \triangleq \begin{bmatrix} \bar{K}_1 & I & -\bar{K}_2 \Theta & \bar{K}_2 & 0 \end{bmatrix},
 \end{aligned}$$

$$\bar{G} \triangleq [G_1^T \bar{C} \quad 0 \quad -G_1^T \quad 0 \quad 0], \quad \Omega_1 \triangleq \begin{bmatrix} \Upsilon_{11} & 0 & \Upsilon_{13} & 0 & 0 \\ * & 0 & 0 & 0 & 0 \\ * & * & \Upsilon_{33} & 0 & 0 \\ * & * & * & -\Lambda_2 & 0 \\ * & * & * & * & \Lambda_3 \end{bmatrix}.$$

According to Assumption 1 and (6), it is obtained that

$$\begin{bmatrix} \bar{e}_k \\ F(\bar{e}_k) \end{bmatrix}^T \begin{bmatrix} \bar{U}_1 & \bar{U}_2 \\ * & I \end{bmatrix} \begin{bmatrix} \bar{e}_k \\ F(\bar{e}_k) \end{bmatrix} \leq 0, \tag{20}$$

and

$$[\bar{\varrho}(\bar{C}\bar{e}_k) - \bar{\Psi}\bar{C}\bar{e}_k]^T [\bar{\varrho}(\bar{C}\bar{e}_k) - \bar{C}\bar{e}_k] \leq 0. \tag{21}$$

By considering (20)-(21) and the triggering condition (7), we have

$$\begin{aligned} & \mathbb{E}\{\Delta J_k\} \\ & \leq \mathbb{E} \left\{ \zeta_k^T (\Omega_1 + \bar{A}^T P \bar{A} + \bar{G}^T \bar{P} \bar{G}) \zeta_k - \mu_1 \begin{bmatrix} \bar{e}_k \\ f(\bar{e}_k) \end{bmatrix}^T \begin{bmatrix} \bar{U}_1 & \bar{U}_2 \\ * & I \end{bmatrix} \begin{bmatrix} \bar{e}_k \\ f(\bar{e}_k) \end{bmatrix} \right. \\ & \quad \left. + \sum_{i=1}^S \mu_2 \left(\frac{1}{\theta_i} s_{i,k} + \sigma_i y_{i,k}^T y_{i,k} - \phi_{i,k}^T \phi_{i,k} \right) - \mu_3 [\bar{\varrho}(\bar{C}\bar{e}_k) - \bar{\Psi}\bar{C}\bar{e}_k]^T [\bar{\varrho}(\bar{C}\bar{e}_k) - \bar{C}\bar{e}_k] \right\} \\ & = \mathbb{E} \left\{ \zeta_k^T (\Omega_1 + \bar{A}^T P \bar{A} + \bar{G}^T \bar{P} \bar{G}) \zeta_k - \mu_1 \begin{bmatrix} \bar{e}_k \\ F(\bar{e}_k) \end{bmatrix}^T \begin{bmatrix} \bar{U}_1 & \bar{U}_2 \\ * & I \end{bmatrix} \begin{bmatrix} \bar{e}_k \\ F(\bar{e}_k) \end{bmatrix} \right. \\ & \quad \left. + \sum_{i=1}^S \frac{\mu_2}{\theta_i} s_{i,k} - \mu_2 \phi_k^T \phi_k + \mu_2 (\Theta \bar{\varrho}(\bar{C}\bar{e}_k) + (I - \Theta) \bar{C}\bar{e}_k)^T \bar{\Lambda}_1 (\Theta \bar{\varrho}(\bar{C}\bar{e}_k) + (I - \Theta) \bar{C}\bar{e}_k) \right. \\ & \quad \left. + \mu_2 \sum_{i=1}^S \bar{\alpha}_i (\Pi_i \bar{\varrho}(\bar{C}\bar{e}_k))^T \bar{\Lambda}_1 (\Pi_i \bar{\varrho}(\bar{C}\bar{e}_k)) + \mu_2 \sum_{i=1}^S \bar{\alpha}_i (\Pi_i \bar{C}\bar{e}_k)^T \bar{\Lambda}_1 (\Pi_i \bar{C}\bar{e}_k) \right. \\ & \quad \left. - 2\mu_2 \sum_{i=1}^S \bar{\alpha}_i (\Pi_i \bar{\varrho}(\bar{C}\bar{e}_k))^T \bar{\Lambda}_1 (\Pi_i \bar{C}\bar{e}_k) - \mu_3 [\bar{\varrho}(\bar{C}\bar{e}_k) - \bar{\Psi}\bar{C}\bar{e}_k]^T [\bar{\varrho}(\bar{C}\bar{e}_k) - \bar{C}\bar{e}_k] \right\} \\ & \leq \mathbb{E} \left\{ \zeta_k^T (\Omega_2 + \bar{A}^T P \bar{A} + \bar{G}^T \bar{P} \bar{G}) \zeta_k \right\} \end{aligned} \tag{22}$$

where

$$\begin{aligned} \tilde{\Upsilon}_{11} & \triangleq -P + \sum_{i=1}^S \bar{\alpha}_i (\Pi_i \bar{C})^T \bar{\Lambda}_1 (\Pi_i \bar{C}) + \bar{C}^T (I - \Theta) \bar{\Lambda}_1 (I - \Theta) \bar{C} - \mu_1 \bar{U}_1 - \mu_3 \bar{C}^T \bar{\Psi}^T \bar{C}, \\ \bar{\Lambda}_1 & \triangleq \text{diag}\{\sigma_1 I, \dots, \sigma_S I\}, \quad \Omega_2 \triangleq \begin{bmatrix} \tilde{\Upsilon}_{11} & -\mu_1 \bar{U}_2 & \tilde{\Upsilon}_{13} & 0 & 0 \\ * & -\mu_1 I & 0 & 0 & 0 \\ * & * & \tilde{\Upsilon}_{33} & 0 & 0 \\ * & * & * & -\bar{\Lambda}_2 & 0 \\ * & * & * & * & \bar{\Lambda}_3 \end{bmatrix}. \end{aligned}$$

By following the similar lines in [17], the exponentially mean-square stability of the augmented system (13) with $\vartheta_k = 0$ can be ensured from (15).

Now, we are in a position to analyze the H_∞ performance of the augmented system (13). For all nonzero ϑ_k , selecting the same Lyapunov function as (16), one has

$$\begin{aligned} & \mathbb{E}\{\Delta J_k\} \\ & = \mathbb{E}\{J_{k+1} - J_k\} \\ & = \mathbb{E} \left\{ \bar{e}_{k+1}^T P \bar{e}_{k+1} - \bar{e}_k^T P \bar{e}_k + \sum_{i=1}^S \frac{1}{\theta_i} (s_{i,k+1} - s_{i,k}) \right\} \\ & \leq \mathbb{E} \left\{ \tilde{\zeta}_k^T (\Omega_3 + \bar{A}^T P \bar{A} + \bar{G}^T \bar{P} \bar{G}) \tilde{\zeta}_k \right\} \end{aligned} \tag{23}$$

where $\bar{\zeta}_k \triangleq [\zeta_k^T \ \vartheta_k^T]^T$ and

$$\Omega_3 \triangleq \begin{bmatrix} \tilde{\Upsilon}_{11} & -\mu_1 \tilde{U}_2 & \tilde{\Upsilon}_{13} & 0 & 0 & \tilde{\Upsilon}_{16} \\ * & -\mu_1 I & 0 & 0 & 0 & 0 \\ * & * & \tilde{\Upsilon}_{33} & 0 & 0 & \tilde{\Upsilon}_{36} \\ * & * & * & -\tilde{\Lambda}_2 & 0 & 0 \\ * & * & * & * & \tilde{\Lambda}_3 & 0 \\ * & * & * & * & * & \bar{D}^T \tilde{\Lambda}_1 \bar{D} \end{bmatrix}.$$

Adding the zero term $\mathbb{E}\left\{\bar{z}_k^T \bar{z}_k - \gamma^2 \vartheta_k^T \vartheta_k - (\bar{z}_k^T \bar{z}_k - \gamma^2 \vartheta_k^T \vartheta_k)\right\}$ to $\mathbb{E}\{\Delta J_k\}$ yields

$$\begin{aligned} & \mathbb{E}\{\Delta J_k\} \\ &= \mathbb{E}\left\{J_{k+1} - J_k + \bar{z}_k^T \bar{z}_k - \gamma^2 \vartheta_k^T \vartheta_k - (\bar{z}_k^T \bar{z}_k - \gamma^2 \vartheta_k^T \vartheta_k)\right\} \\ &\leq \mathbb{E}\left\{\bar{\zeta}_k^T (\Omega_4 + \bar{A}^T P \bar{A} + \bar{G}^T \bar{P} \bar{G}) \bar{\zeta}_k\right\} - \mathbb{E}\left\{\bar{z}_k^T \bar{z}_k - \gamma^2 \vartheta_k^T \vartheta_k\right\}. \end{aligned} \tag{24}$$

Under the zero-initial condition, summing up (24) on both sides from 0 to T with respect to k leads to

$$\begin{aligned} & \sum_{k=0}^T \mathbb{E}\{\Delta J_k\} \\ &\leq \sum_{k=0}^T \mathbb{E}\left\{\bar{\zeta}_k^T (\Omega_4 + \bar{A}^T P \bar{A} + \bar{G}^T \bar{P} \bar{G}) \bar{\zeta}_k\right\} - \sum_{k=0}^T \mathbb{E}\left\{\bar{z}_k^T \bar{z}_k\right\} + \sum_{k=0}^T \gamma^2 \vartheta_k^T \vartheta_k \end{aligned} \tag{25}$$

and hence

$$\begin{aligned} & \mathbb{E}\left\{\sum_{k=0}^T \|\bar{z}_k\|^2\right\} - \gamma^2 \sum_{k=0}^T \|\vartheta_k\|^2 \\ &\leq \sum_{k=0}^T \mathbb{E}\left\{\bar{\zeta}_k^T (\Omega_4 + \bar{A}^T P \bar{A} + \bar{G}^T \bar{P} \bar{G}) \bar{\zeta}_k\right\} - \mathbb{E}\{J_{T+1}\} \\ &\leq \sum_{k=0}^T \mathbb{E}\left\{\bar{\zeta}_k^T (\Omega_4 + \bar{A}^T P \bar{A} + \bar{G}^T \bar{P} \bar{G}) \bar{\zeta}_k\right\}. \end{aligned} \tag{26}$$

Noting (15) and letting $T \rightarrow +\infty$, we can easily conclude that the H_∞ performance constraint (14) is met, which completes this proof. \square

In terms of the obtained condition in Theorem 1, the algorithm for designing the estimator parameters is provided in the following theorem.

Theorem 2. Let the disturbance level γ be given. The H_∞ non-fragile state estimation problem is solvable for CNs (1) if there exist positive matrices $P \triangleq \text{diag}\{P_1, P_2\}$, $P_2 \triangleq \text{diag}\{P_{21}, \dots, P_{25}\}$, matrix $X \triangleq \text{diag}\{X_1, \dots, X_5\}$ and five positive scalars $\{\mu_j\}_{1 \leq j \leq 5}$ satisfying the following LMI:

$$\bar{\Omega} \triangleq \begin{bmatrix} \Omega_4 & \tilde{\Omega}_4 & \Omega_5 & 0 \\ * & \Omega_6 & 0 & \Omega_7 \\ * & * & \Omega_8 & 0 \\ * & * & * & \Omega_8 \end{bmatrix} < 0 \tag{27}$$

where

$$\begin{aligned} & \tilde{\Omega}_4 \triangleq [\check{G}^T \ \bar{A}^T], \ \Omega_6 \triangleq \text{diag}\{-\bar{P}, -P\}, \ \check{A} \triangleq [\check{K}_1 \ P \ -\check{K}_2 \Theta \ \check{K}_2 \ 0 \ \check{B}], \ \bar{M} \triangleq \text{diag}_{2S}\{M\}, \\ & \check{G} \triangleq [\check{G}_1^T \check{C} \ 0 \ -\check{G}_1 \ 0 \ 0 \ 0], \ \bar{M} \triangleq \text{diag}_2\{M\}, \ \Omega_7 \triangleq \text{diag}\{\bar{P} \bar{M}, P \bar{M}\}, \ \Omega_8 \triangleq \text{diag}\{-\mu_4 I, -\mu_5 I\}, \\ & \Omega_5 \triangleq \begin{bmatrix} \mu_4 \hat{G}_2^T \check{C} & 0 & -\mu_4 \hat{G}_2^T & 0 & 0 & 0 \\ \mu_5 \tilde{A}_2 & 0 & -\mu_5 \tilde{A}_3 \Theta & \mu_5 \tilde{A}_3 & 0 & \mu_5 \tilde{A}_4 \end{bmatrix}^T, \ \check{K}_1 \triangleq \begin{bmatrix} P_1(W \otimes \Gamma) & 0 \\ X \Theta C & P_2(W \otimes \Gamma) - XC \end{bmatrix}, \\ & \tilde{A}_2 \triangleq \begin{bmatrix} 0 & 0 \\ N \Theta C & -NC \end{bmatrix}, \ \tilde{A}_4 \triangleq \begin{bmatrix} 0 & 0 \\ 0 & -ND \end{bmatrix}, \ \check{K}_2 \triangleq \begin{bmatrix} 0 \\ X \end{bmatrix}, \ \check{B} \triangleq \begin{bmatrix} P_1 B & 0 \\ P_2 B & -XD \end{bmatrix}, \ \tilde{A}_3 \triangleq \begin{bmatrix} 0 \\ N \end{bmatrix}, \\ & \check{G}_1 \triangleq [\sqrt{\alpha_1} \Pi_1 \check{K}_2^T \ \dots \ \sqrt{\alpha_5} \Pi_5 \check{K}_2^T], \ \hat{G}_2 \triangleq [\sqrt{\alpha_1} \Pi_1 \tilde{A}_3^T \ \dots \ \sqrt{\alpha_5} \Pi_5 \tilde{A}_3^T]. \end{aligned}$$

Furthermore, if the LMI (27) is feasible, the desired estimator parameters are determined by

$$K = P_2^{-1} X. \tag{28}$$

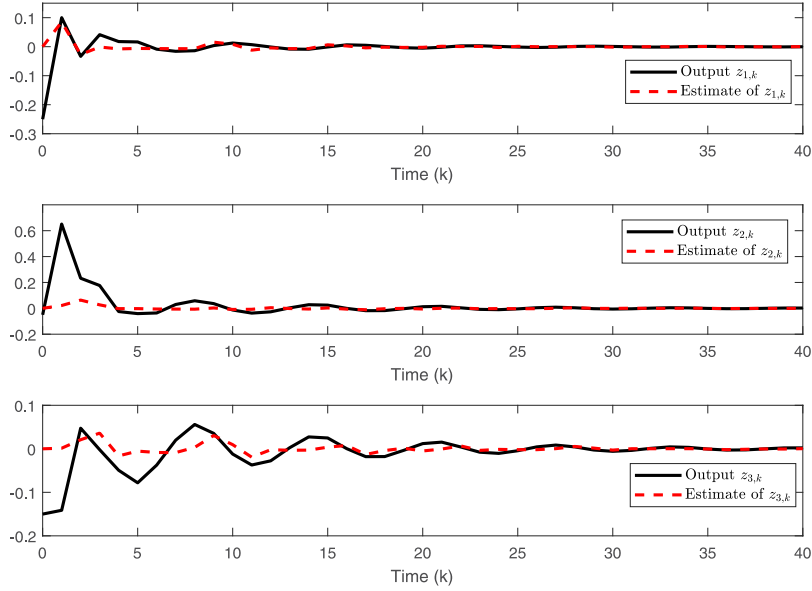


Fig. 1. Output $z_{i,k}$ ($i = 1, 2, 3$) and their estimate.

Proof. The matrix $\tilde{\Omega}$ can be decomposed as follows:

$$\tilde{\Omega} = \tilde{\Omega}_1 + \tilde{\Omega}_2, \quad (29)$$

where $\tilde{\Omega}_2 \triangleq \Psi_1^T \bar{F}_k \Psi_2 + (\Psi_1^T \bar{F}_k \Psi_2)^T + \Psi_3^T \bar{F}_k \Psi_4 + (\Psi_3^T \bar{F}_k \Psi_4)^T$ and

$$\begin{aligned} \tilde{\Omega}_1 &\triangleq \begin{bmatrix} \Omega_4 & \hat{G}^T \bar{P} & \hat{A}^T P \\ * & -\bar{P} & 0 \\ * & * & -P \end{bmatrix}, \bar{F}_k \triangleq \text{diag}_2\{F_k\}, \hat{A} \triangleq [\hat{K}_1 \quad I \quad -\hat{K}_2 \Theta \quad \hat{K}_2 \quad 0 \quad \hat{B}], \\ \hat{G} &\triangleq [\hat{G}_1^T \bar{C}_k \quad 0 \quad -\hat{G}_1^T \quad 0 \quad 0 \quad 0], \hat{K}_1 \triangleq \begin{bmatrix} W \otimes \Gamma & 0 \\ K \Theta C & W \otimes \Gamma - KC \end{bmatrix}, \hat{K}_2 \triangleq \begin{bmatrix} 0 \\ K \end{bmatrix}, \\ \hat{G}_1 &\triangleq [\sqrt{\alpha_1} \Pi_1 \hat{K}_2^T \quad \dots \quad \sqrt{\alpha_5} \Pi_5 \hat{K}_2^T], \hat{B} \triangleq \begin{bmatrix} B & 0 \\ B & -KD \end{bmatrix}, \bar{F}_k \triangleq \text{diag}_{25}\{F_k\}, \\ \Psi_1 &\triangleq [\hat{G}_2^T \bar{C} \quad 0 \quad -\hat{G}_2^T \quad 0 \quad 0 \quad 0 \quad 0 \quad 0], \Psi_2 \triangleq [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \bar{M}^T \bar{P} \quad 0], \\ \Psi_3 &\triangleq [\tilde{A}_2 \quad 0 \quad -\tilde{A}_3 \Theta \quad \tilde{A}_3 \quad 0 \quad \tilde{A}_4 \quad 0 \quad 0], \Psi_4 \triangleq [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \bar{M}^T P]. \end{aligned}$$

According to (15), one has

$$\tilde{\Omega} = \tilde{\Omega}_1 + \Psi_1^T \bar{F}_k \Psi_2 + (\Psi_1^T \bar{F}_k \Psi_2)^T + \Psi_3^T \bar{F}_k \Psi_4 + (\Psi_3^T \bar{F}_k \Psi_4)^T \leq 0. \quad (30)$$

It follows from Lemma 2 that (30) is true if there exist positive scalars μ_4 and μ_5 such that

$$\tilde{\Omega}_1 + \mu_4^{-1} \Psi_2^T \Psi_2 + \mu_4 \Psi_1^T \Psi_1 + \mu_5^{-1} \Psi_4^T \Psi_4 + \mu_5 \Psi_3^T \Psi_3 \leq 0. \quad (31)$$

Furthermore, by noting $KP_2 = X$, we conclude that inequality (31) is satisfied if (27) holds. This completes the proof. \square

Remark 3. So far, we have dealt with the non-fragile H_∞ state estimation problem for a class of discrete-time CNs under DETMs and ROSSs. In Theorem 1, a sufficient condition has been established to ensure the exponentially mean-square stability and H_∞ constraint of the estimation error dynamics. Moreover, in Theorem 2, the explicit expression of the estimator gain matrix has been obtained in terms of the solutions to the LMI (27). It is observed from Theorems 1–2 that all the information about the considered phenomena (e.g. nonlinearities, estimator gain variations, DETMs, ROSSs and external disturbances) have been reflected.

4. Illustrative examples

In this section, a numerical example is provided to show the effectiveness of the addressed non-fragile estimation methods of this paper.

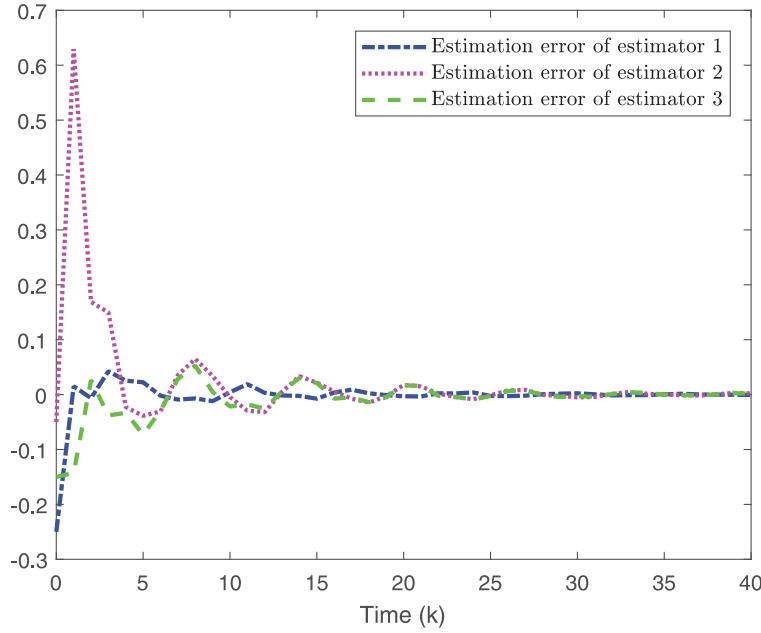


Fig. 2. Estimation errors $\tilde{z}_{i,k}$ ($i = 1, 2, 3$).

Consider a CN (1) consisting of 3 nodes with measurement output (3), which have the following parameters:

$$B_1 = \begin{bmatrix} -0.4 \\ 0.2 \end{bmatrix}, B_2 = \begin{bmatrix} 0.5 \\ 0.6 \end{bmatrix}, B_3 = \begin{bmatrix} 0.2 \\ 0.6 \end{bmatrix}, C_1 = [1 \quad 0.7], C_2 = [1.5 \quad 0.9],$$

$$E_1 = [-0.4 \quad 0.2], E_2 = [0.5 \quad 0.6], E_3 = [0.2 \quad 0.6], C_3 = [0.8 \quad 1.6], \Gamma = \text{diag}\{0.1, 0.1\},$$

$$D_1 = 0.15, D_2 = 0.5, D_3 = 0.35, U_1 = \begin{bmatrix} -0.6 & 0.3 \\ 0 & 0.4 \end{bmatrix}, U_2 = \begin{bmatrix} -0.3 & 0.3 \\ 0 & 0.6 \end{bmatrix}, \omega_{ij} = \begin{cases} -0.2, & i = j, \\ 0.1, & i \neq j. \end{cases}$$

The nonlinear function is chosen as $f(x_{i,k}) = 0.5((U_1 + U_2)x_{i,k} + (U_2 - U_1)\sin(k)x_{i,k})$.

The parameters of the estimator gain variations are given as follows:

$$M_1 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, M_2 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, M_3 = \begin{bmatrix} 0.12 \\ -0.1 \end{bmatrix}, N_1 = 0.1, N_2 = -0.1, N_3 = 0.12, F_{1,k} = F_{2,k} = F_{3,k} = \sin(k).$$

For the DETM (7) with (8), set $\lambda_1 = \lambda_2 = \lambda_3 = 0.8$ and $\theta_1 = \theta_2 = \theta_3 = 2$ to satisfy Lemma 1, and select the thresholds as $\sigma_1 = \sigma_2 = \sigma_3 = 0.5$. The probabilities α_i ($s = 1, 2, 3$) are taken as 0.5, 0.7 and 0.3, respectively. The sensor saturation parameters are considered as $\beta_i = 0.1$ ($i = 1, 2, 3$).

By using the Matlab toolbox, we solve the LMI (27) to obtain the following solutions:

$$P_{21} = \begin{bmatrix} 1.9641 & -1.4901 \\ -1.4901 & 2.0281 \end{bmatrix}, X_1 = \begin{bmatrix} -0.3922 \\ 0.4528 \end{bmatrix}, P_{22} = \begin{bmatrix} 1.8156 & -1.0650 \\ -1.0650 & 3.2845 \end{bmatrix},$$

$$X_2 = \begin{bmatrix} -0.1111 \\ 0.2496 \end{bmatrix}, P_{23} = \begin{bmatrix} 0.8566 & -1.0226 \\ -1.0226 & 3.7267 \end{bmatrix}, X_3 = \begin{bmatrix} -0.2818 \\ 0.6164 \end{bmatrix}.$$

Then, according to (28), we have

$$K_1 = \begin{bmatrix} -0.0685 \\ 0.1730 \end{bmatrix}, K_2 = \begin{bmatrix} -0.0205 \\ 0.0694 \end{bmatrix}, K_3 = \begin{bmatrix} -0.1955 \\ 0.1117 \end{bmatrix}.$$

In this example, the initial values are chosen as

$$x_{1,0} = [1 \quad 0.75]^T, x_{2,0} = [-1 \quad 0.75]^T, x_{3,0} = [0.75 \quad -0.5]^T, \varsigma_{1,0} = \varsigma_{2,0} = \varsigma_{3,0} = 0.$$

Furthermore, we set $\gamma = 0.8$, $w_k = \sin(k)\exp(-0.1k)$ and $v_k = 2\cos(2k)\exp(-0.1k)$.

In the simulation, Fig. 1 plots the output and their estimates at each node of the underlying CNs. Fig. 2 presents the estimation errors. The triggering instants of measurement outputs are described in Fig. 3. For node i ($i = 1, 2, 3$), Fig. 4 shows

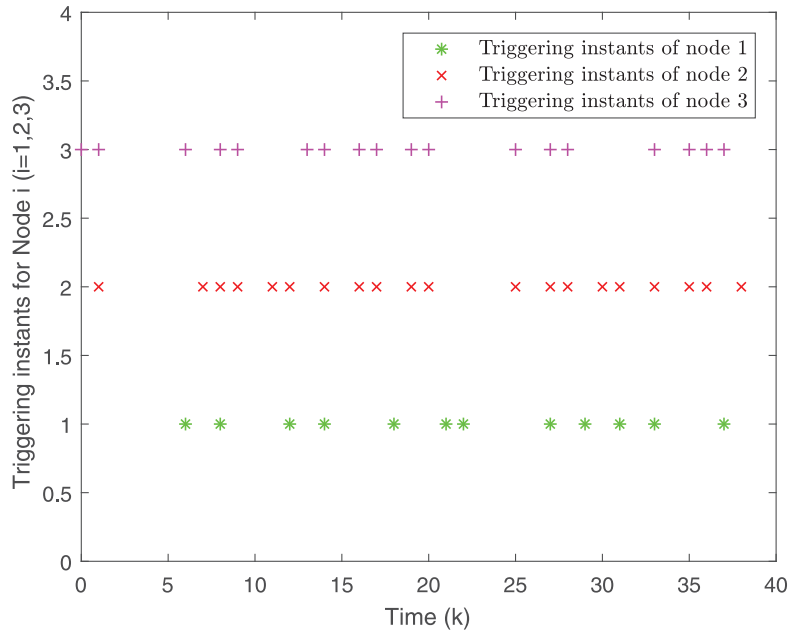


Fig. 3. The triggering instants.

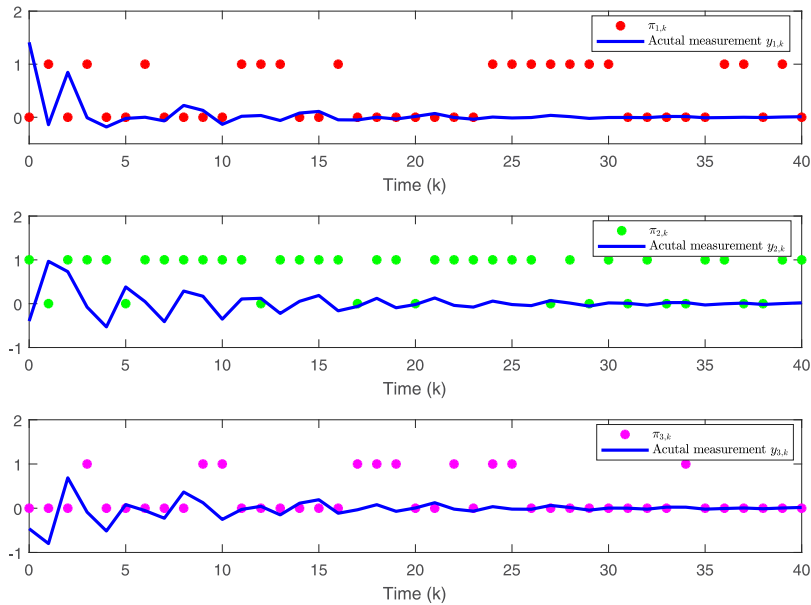


Fig. 4. Measurements from node i ($i = 1, 2, 3$).

the actual measurement $y_{i,k}$ and the binary signal $\pi_{i,k}$. The simulation results have confirmed the effectiveness of the proposed non-fragile H_∞ estimation algorithm.

5. Conclusions

This paper has investigated the dynamic event-triggered H_∞ state estimation for a class of discrete-time CNs with estimator gain perturbations and ROSSs. In order to save energy, the DETM has been employed in the transmission between the sensor and its corresponding state estimator. By constructing a novel Lyapunov function, a sufficient condition has been established under which the estimation error dynamics is exponentially mean-square stable with the given H_∞ performance level. Then, the parameters of the desired estimator have been obtained in terms of the solution to a LMI. Finally, the useful-

ness of the estimator design algorithm proposed has been confirmed via a simulation example. One of our future research topics would be to investigate the fault estimation problem of CNs under DETMs by using intelligent methods [4,6,39].

Declaration of Competing Interest

None.

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